



$$\int_S \eta (\nabla \mathbf{u}^T) \cdot \mathbf{n} dS = \int_S \left(\eta \frac{\partial u_j}{\partial x_i} \mathbf{i}_i \mathbf{i}_j \right) \cdot (n_k \mathbf{i}_k) dS$$

$$= \int_S \eta \frac{\partial u_j}{\partial x_i} n_k \mathbf{i}_i \mathbf{i}_j \mathbf{i}_k dS = \int_S \eta \frac{\partial u_j}{\partial x_i} n_k \mathbf{i}_i \delta_{jk} dS = \int_S \eta \frac{\partial u_j}{\partial x_i} n_j \mathbf{i}_i dS = \int_S \eta \left(\frac{\partial u_1}{\partial x_i} n_1 + \frac{\partial u_2}{\partial x_i} n_2 \right) \mathbf{i}_i dS$$

当 $i=1$ 时，即对应于 x 方向的动量方程时，同时在如上图所示结构化网格中离散，记 $u_1 = u, u_2 = v$ ，上述方程可简化为：

$$= \int_S \left(\eta \frac{\partial u}{\partial x} n_1 + \eta \frac{\partial v}{\partial x} n_2 \right) \mathbf{i} dS$$

$$= \left[\left(\eta \frac{\partial u}{\partial x} \right)_e S_e - \left(\eta \frac{\partial u}{\partial x} \right)_w S_w + \left(\eta \frac{\partial v}{\partial x} \right)_n S_n - \left(\eta \frac{\partial v}{\partial x} \right)_s S_s \right] \mathbf{i}$$

$$= \left[\eta_e S_e \frac{u_E - u_P}{(\delta x)_e} - \eta_w S_w \frac{u_P - u_W}{(\delta x)_w} + ? + ? \right] \mathbf{i}$$

后面两项我实在是不知道怎么离散，是

$$\eta_n S_n \left[\frac{v_{NE} + v_E}{2} - \frac{v_{NW} + v_W}{2} \right] \frac{1}{(\delta x)_w + (\delta x)_e} - \eta_s S_s \left[\frac{v_{SE} + v_E}{2} - \frac{v_{SW} + v_W}{2} \right] \frac{1}{(\delta x)_w + (\delta x)_e}$$

吗？我实在不敢确定，希望各位前辈指点一点

同理， $i=2$ 时对应于 y 方向的动量方程：

$$= \int_S \left(\eta \frac{\partial u}{\partial y} n_1 + \eta \frac{\partial v}{\partial y} n_2 \right) \mathbf{j} dS \text{ 也同样存在相同的问题，求高人指点。}$$