$$\frac{\partial \alpha}{\partial t} + \frac{\partial (u_d \alpha)}{\partial x} = 0$$

$$\frac{\partial \alpha}{\partial t} + u_d \frac{\partial \alpha}{\partial x} + \alpha \frac{\partial u_d}{\partial x} = 0$$

if $\frac{\partial u_d}{\partial x} = 0$

$$\frac{\partial \alpha}{\partial t} + u_d \frac{\partial \alpha}{\partial x} = 0 \rightarrow \text{wave equation}$$

↓

bounded.

if $\frac{\partial u_d}{\partial x} \neq 0$

$$\frac{\partial \alpha}{\partial t} + u_d \frac{\partial \alpha}{\partial x} = \cancel{\alpha \frac{\partial u_d}{\partial x}} - \alpha \frac{\partial u_d}{\partial x}$$

↓

unbounded.

— Dongwe Li

2018.01.29.